

Message transmission by means of counting states of sets

Steidl, R. & Javorszky, K.

Institut fuer angewandte Statistik,
Landhausgasse 4/23, Austria. Tel./Fax + 43 1 533 47 40 (kj.sci@chello.at)

Abstract

A new method of concurrent processing of discrete states of media may be useful in messages transmission. The method uses multichannel processing of signals. The signals are understood to signify distinct logical states of sets. The method works by messages being encoded and deciphered by symbols being present on a multitude of carriers. The carriers may be sent in bulk (no sequential order among carriers is maintained).

By using implicit agreements between sender and receiver, the message object can be identified by interpreting the message as a description of a distinct partitional state of a set. The implicit agreements relate to the collection of most probable partitional states of sets.

Concurrently received descriptions which refer to an identical message object allow probabilistic decisions about the "nature" of the message object in question.

The method can be useful in approximating the efficiency of biologic information processing mechanisms ("sensory organs"). The tools used may even be useful in providing a mathematical model of the structure of abstract assemblies.

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Contents

1. INTRODUCTION	3
2. ONEDIMENSIONAL DESCRIPTIONS OF SETS, THE STRUCTURE OF A SET	3
2.1. Transmission procedure	3
2.2. An example of enumeration of states of a set	4
2.3. Using 2 distinct symbols to generate mutually exclusive subsets	5
2.4. General formulation of the concept	5
2.5. Discussion of the equivalence of partitions to states of sets	5
2.5.1. 1st iteration, using 1 - the same - symbol to be attached to each object	6
2.5.2. 2nd iteration, using 2 symbols	7
2.5.3. n-th iteration, using n symbols	7
2.6. Semantic interpretation of the concept "state of a set"	8
2.7. Number of logical symbols on elements of a set and number of summands of a sum	8
2.8. Probable and most probable states of a set	9
3. MULTIDIMENSIONAL DESCRIPTIONS OF SETS, CODING AND DECODING OF MESSAGES	10
3.1. Structures of levels 1 and 2	10
3.1.1. First symbols	10
3.1.2. Second symbol	10
3.1.3. Redundant symbols	11
3.1.4. Negated symbols	11
3.2. Identifying distinct messages	11
3.3. Ease and efficiency of distinguishing distinct messages	12
3.4. Practical procedure of coding and decoding	14
4. RELEVANCE OF THE MODEL TO BIOLOGIC INFORMATION PROCESSING	14
4.1. Transmission efficiency of pointing out improbable properties	14
4.2. Differentiation between "message objects" and "messages"	15
4.3. Existence of upper limit for structures on sets	15
5. DISADVANTAGES AND ADVANTAGES OF TRANSMITTING MESSAGES BY USING GROUP STRUCTURES ON SETS	16
5.1. Disadvantages	16
5.2. Advantages	16
6. SUMMARY, OUTLOOK	17

1. Introduction

The pattern recognition mechanisms in biology do not allow assumptions of signal processing by the von Neumann method. Nor does the architecture of sensory cells lend itself easily to the idea that points in an Euclidean space can be identified by means of orthogonal, independent axes.

The methods biology uses deviate at least in following respects to concepts of classical information theory:

- several receiving channels cooperate,
- the transmission happens concurrently,
- a message is realised on a multitude of carriers,
- the specific nature of the sensory input is irrelevant,
- the signals having been input remain in the processing black box for a while,
- the message is compared to a previous, or to a most usual, message.

We have undertaken the task of simulating perception by developing a coding-decoding algorithm which, as far as we succeeded in the task, imitates the above system characteristics.

In order to conceptualise a mathematical model of biologic information processing we have evolved the following basic assumptions:

- logical relations among elements of the set exist;
- the symbols used to denote logical relation(s) among elements of a set are in their ontologic nature indistinguishable, not fixed;
- states of sets exist;
- differing states of sets may have different probabilities;
- most probable states of sets exist;
- there exists a most probable distribution of symbols on a set.

Our concept uses the most probable distribution of symbols on sets being in their most probable state as a background, in comparison to which each individual message is to be understood/deciphered.

In this paper we first introduce the terms by means of examples, then we discuss concepts of probability, relating to onedimensional states of sets, then we generalise the concept of states of sets into more describing dimensions. We close by giving our opinion on possible advantages and disadvantages of using the technique presented here.

2. Onedimensional descriptions of sets, the structure of a set

We introduce the concepts by means of a transmission example, where a Sender sends messages to a Receiver by means of things carrying signs.

2.1. Transmission procedure

A Sender sends a collection of things to a Receiver. On some of the things Sender has placed a symbol. What kind of things and what kind of symbols will not be discussed. We discuss, how many distinct group relations can Sender generate and Receiver perceive if the message pouch contains n

objects. Sender uses n indistinguishable carriers, called objects (e.g. white pieces of paper). ($n > 3$, $n \ll \infty$). Sender picks any of an indeterminate number of distinct symbols (e.g. crosses, black dots, circles, letters of the alphabet, etc.). Sender marks some of the objects with symbol(s) and sends the collection to Receiver.

Receiver can distinguish as many distinct messages as distinct logical states are discernible on the set of carriers in a POUCH which Sender sends to Receiver.

It is important that the sequential order of the media is not relevant. The pieces of paper (or the billiard balls) can be shuffled in any sequence. The nature of the symbol employed does not come into discussion.

Sender marks a number n_1 of objects with a symbol S_1 . In practice, Sender draws a cross (or a circle or the letter W or the letter M etc.) on up to the half of the previously white pieces of paper (etches a symbol on billiard balls, etc.)

A symbol which is present on more than half of the objects may or may not be substituted by its negation. Receiver may say "I see on 4 of the 5 billiard balls a black dot" or he may say "I see on 1 of the 5 billiard balls a dotless special spot". Which apperceptual method he shall choose is his own choice and Sender shall assume that a symbol being present on more than $n/2$ of n carriers has the identical communicational value as its negation being present on the remaining objects.

A symbol which is present on ALL objects shall not be perceived by the receiver. If Sender marks all pieces of white paper with a yellow paint, Receiver shall believe that he will have to decipher a message coded on yellow paper. If Sender flattens all billiard balls into the shape of cubes, Receiver shall believe that he is being communicated by means of media which are cubes (and not billiard balls).

The distinct states of a collection of objects are a combination of distinct states of individual objects. The collection of the distinct states of the objects makes up a state of the collection.

Having put a distinguishing symbol on some objects, Sender has created a distinct state of the collection of objects. In the simplest case, Sender creates two groups of objects: group I includes objects with no symbols on them and group II includes objects which have a symbol on them.

2.2. An example of enumeration of states of a set

In the simplest case, sender marks exactly k objects out of n objects, and he marks each of the k objects with the same symbol (whichever symbol that may be). ($0 \leq k \leq n$)

An enumeration of the states of the set is up to $k/2$ bijectively mappable to elements of N , namely e.g. as follows:

No of elements marked	Nr. of state	Nr. of message
0	0	1.1
1	1	1.2
2	2	1.3
...+1

We see that we transmit messages by an agreed upon - implicit - enumeration of distinct states of the set. As many distinct states of the set we are able to discern, as many distinct messages we are able to transmit.

2.3. Using 2 distinct symbols to generate mutually exclusive subsets

Sender may use different symbols to transmit messages. Let us proceed in minimal steps and discuss the case that sender uses any of 2 distinct symbols and places on each element up to 1 of the 2 symbols but never both. We see that each object of the the collection falls into one of the following 3 possible categories:

- a) empty object (no symbols, neither S1 nor S2),
- b) object with symbol S1
- c) object with symbol S2.

To simplify matters more, let us assume that each element does carry one of the two possible symbols S1 and S2. Otherwise, we would discuss trisection of the set before bisection.

Bisecting the set into objects which carry symbol S1 and into objects which carry symbol S2 allows us to introduce the concept of the state of a set. A state of a set: On the set we have conducted a partition. Objects with an identical symbol S_i have been grouped together. As all objects carry exactly one symbol, they shall be grouped into exactly one subset. Thus we observe a partition of the set.

So far, a structure of level 1 has been imposed on the set. There is 1 describing dimension relating to the set, namely the existence or nonexistence of the symbol on elements of the set. There is exactly one symbol on each object. The symbol generates mutually exclusive groups of elements, namely those which carry the symbol S1 and those which carry symbol S2.

2.4. General formulation of the concept

The state of a set is the collection of logical operations conducted on the set.

A logical operation is the application of receiver-distinct symbols to objects of a set.

Each of the logical operations generates distinct subsets of the set which in their union make up the set. Thus each of the logical operations is a partition of the set.

The collection of logical operations conducted on the set may create collections of subsets which are not pairwise disjoint.

2.5. Discussion of the equivalence of partitions to states of sets

We shall now discuss in more general terms the idea that several distinct symbols are in use. The symbols shall at first generate disjoint subsets.

The set of partitions $E(n)$ of an n from N is the collection of the mutually exclusive and exhaustive elements of onedimensional partitions of the natural number n .

E.g. for $n = 5$ this set of partitions $E(n)$ is:

{ (5), (4,1), (3,2), (3,1,1), (2,2,1), (2,1,1,1), (1,1,1,1,1) }

Let $\{U\}$ be a collection of objects. Let $\text{card}(U) = n$. ($n > 3$, $n \ll \text{inf}$)

The cardinality n of the finite set U is represented by a map to N , by the natural number n . In respect to one describing dimension, the set can be in $E(n)$ distinct states, where $E(n)$ denotes the number of partitions of the natural number n .

Let $\{S\}$ be a collection of symbols. Let $\text{card}(S) > n$.

Let $S1$ be an element of the collection $\{S\}$. Logical operation "a" assigns $S1$ to each of the unit summands of n . This operation is equivalent to operation "a", conducted on set U .

We shall now discuss logical operation "a" by going through iterations. First we shall assign the same symbol to each element of U and concurrently to each unit element of N from the collection of 1-s which make up n . In the 2nd iteration we shall use 2 symbols and show that we can generate as many distinct states of set U as there are partitions of n into 2 summands. The last iteration we discuss here shall assign to each element of U a distinct symbol and concurrently to each unit summand of n a distinct symbol.

2.5.1. 1st iteration, using 1 - the same - symbol to be attached to each object

The first iteration assigns to each element of the set the same symbol. This has been referred to above as message Nr. 1.1. We group all (each) element(s) of U together by applying to all (each) of them the same symbol. This symbol is redundant, as it appears on all (each) element(s). We go through this iteration for didactic purposes.

A description of U with respect of its state is equivalent to counting the units of N which possess the selection criterium "a". In both cases - approaches -, the logical operation yields one result, namely U resp. n . We lump together those elements of U which are indistinguishable with respect to $S1$ and state that each element of U is a subset of the resulting set, that is, the set U is identical to the set which we observe if we throw all those elements of U together which carry the symbol $S1$.

We take each element of U which has been marked with symbol $S1$ and see that we have n such elements. We count: "1st element: yes, belongs here, add 1"; "2nd element: yes, belongs here, add 1"; "3rd element: yes, belongs here, add 1"; ...; "n-1st element: yes, belongs here, add 1" and "nth element: yes, belongs here, add 1". Thus we have said "add 1" n times, that is, the result is n .

Exemplification of the term "logical sentence":

We have said n times a sentence of the form "{t.|f.},{operation on set}, {operation on N }". A sentence which states a logical constant and/or a logical operation (including the operations on N which are considered to be logical operations) is termed a logical sentence.

The description of U by means of logical sentences has happened n times identically. We lump the descriptive sentences together and say that we have n identical sentences.

Thus we observe a specific state having been generated by attaching symbol $S1$ to each element of Set U and describe this state by the partition of n into 1 category (subset).

We have only one way of observing n times unit inclusion into a set with identical logical symbols on the elements. This is denoted by the expression $E(n,1) = 1$. The expression $E(n,k)$ refers to the set of partitions of an n into exactly k summands. E.g. $E(5,1) = 1$, namely $\{(5)\}$ and $E(4,2) = 2$, namely $\{(3,1), (2,2)\}$.

$E(n,1) = 1$ says that there is only one way of observing 1 symbol on all n units.

As a message, this state of the set has been enumerated as message Nr. 1.1. (All objects carry either no discernible symbol at all or a symbol which is common to each of them.)

2.5.2. 2nd iteration, using 2 symbols

We now pick 2 symbols from the set S , namely $S1$ and $S2$. We can generate distinct states of U by means of 2 distinct symbols in exactly $E(n,2)$ ways, namely in as many ways as we have writing n as a sum of 2 summands. For $n = 5$, there are 2 ways of writing n as a sum of 2 summands, namely $5 = 4 + 1$ and $5 = 3 + 2$. We say $E(5,2) = 2$.

As a procedure conducted on a set, we take $q = 1..n-1$ elements and assign $S1$ to q elements. To the remaining $n-q$ elements we assign $S2$. Obviously, $\#(S1) + \#(S2) = n$. Each element carries exactly one sign.

Each of the describing sentences which state "There are q elements with one of the symbols $\{S1 | S2\}$ and there are $n-q$ elements with the other symbol" describes a state of the set, a partition. There can be as many distinct states of a set of n elements where each element carries one of two distinct logical symbols as there are partitions of n into 2.

For $q = 1$, we know that 1 of the elements carries a symbol distinct to the symbol each of the other elements carries. It is irrelevant, which of the elements is distinguished against the other $n-1$ elements. Further above, this state of the set has been introduced as message 1.1.

For $q = 2$, we know (and can recognise by regarding white pieces of paper or billiard balls) that 2 of the elements of U carry a symbol which is distinct to the symbol which $n-2$ elements carry.

The operations on N map the state changes of the set. Using 2 symbols, we can generate $E(n,2)$ distinct additive sentences. Each of the sentences says: " $n1 + n2 = n$ ", which is a different way of saying "The state of the set with n elements of which each carries one of two symbols is such, that $n1$ of the elements carry symbol 'a' and $n2$ of the elements carry symbol 'b'. The collection of elements is the set. The set can be subdivided into two mutually exclusive subsets, of which the cardinalities are, respectively, $n1$ and $n2$, where $n1$ denotes the number of elements which carry one symbol, and $n2$ is the number of elements which carry the other symbol."

We skip the inductively obvious halfsteps and discuss

2.5.3. n-th iteration, using n symbols

We pick of the collection $\{S\}$ of symbols n distinct symbols and attach one symbol each to one distinct element of the set. There is only one way of doing this. We consider those elements which carry no symbol so far as indistinct and remove those elements which do already have a symbol attached to them from the set of the unmarked elements.

There is only one way of formulating as an additive sentence what we have done, namely " $1 + 1 + 1 + \dots + 1 = n$ ". We say $E(n,n) = 1$. This description of the state of the set is to be translated into natural language e.g. as follows:

"We observe one element which is unique and call this the 1st element. Then we observe one element which is unique and call this the 2nd element. We continue observing one element at a time

which is unique and enumerate the observations consecutively. We shall have said altogether n times "here we have a unique element". There is only one way of assigning a distinct symbol to each of the elements."

2.6. Semantic interpretation of the concept "state of a set"

According to definitions of set theory, a set is a collection of objects which share a common property. Let us consider as an example the set of patients of a hospital ward. Let there be n patients in the course of a day. If the set is in its usual state, n_1 of the patients shall be in a state called "critical", n_2 in a state called "febril", n_3 in a state called "unclear" and n_4 in a state called "reconvalescent".

A state change is observed, if one of the patients shows an improved or a worsened condition. Let at a time T_0 the status report of the station show $n_1 + n_2 + n_3 + n_4 = n$ with fixed n_i and n . Let us observe at a time T_1 that something happened and that almost every patient is now in a state called "critical". The total number of patients shall remain the same. ($n_{T_0} = n_{T_1}$) The description of the state will still be formally $\text{Sum}(n_i) = n$, but we shall see differing n_1 values ($n_{1T_0} \neq n_{1T_1}$), correspondingly differing n_2 , n_3 and/or n_4 values as well.

That changes of states of sets have a great everyday importance is patently obvious. The symbols attached to elements of sets constitute groups of elements. In case the groups are mutually exclusive, we speak of a one-dimensional logical description of the state of the set. As the groups are exclusive and their cardinalities additive, we use the partitions to describe one-dimensional structures on sets.

2.7. Number of logical symbols on elements of a set and number of summands of a sum

Regarding a one-dimensional description of the state of a set, one can draw farreaching parallels between the logical operations connecting elements of a set and of the numeric operation known as addition conducted on elements of N .

A one-dimensional structure establishes exclusive groups within the set. An addition establishes one-dimensional concatenations between intervals on x , the numeric representation of N .

As many distinct symbols are present on a set, as many subsets can be used to group the objects into. We do not take into account, which elements are distinguished by the symbols.

If we regard exactly one symbol per element of the set, we can state that U includes $U_1, U_2, U_3, \dots, U_k$. The distinct symbols $S_1, S_2, S_3, \dots, S_k$ have generated collections of elements which are distinguished by carrying symbol S_1, S_2, \dots, S_k .

The cardinalities are in this order: $n = n_1 + n_2 + n_3 + \dots + n_k$.

2.8. Probable and most probable states of a set

Now we want to use the well-known partitional properties of numbers to discuss most probable states of a set.

The set of partitions $E(n)$ is a probability body.

A probability body is a collection of distinct occurrences which are mutually exclusive and altogether exhaustive. Each partition of n is an occurrence describing a distinct state of a set. The occurrences are distinct, exhaustive und exclusive.

A numeric approach to the probability concept is usually connected - by tradition - to such a representation in R , where the totality of the occurrences is treated as 1.0. To achieve this conformity, we treat the numerosity of the set $E(n)$ as Unit (1.0) and norm the individual partitions to this extent. (In practice, the probability of such a state of the set to exist as is described by a specific partition is given by the relative frequency of this partition among all partitions.

E.g. on the set $\{ (5), (4,1), (3,2), (3,1,1), (2,2,1), (2,1,1,1), (1,1,1,1,1) \}$, which is the set of onedimensional partitions of 5,

the probability is	for
1/7	any specific onedimensional partitional state,
1/7	states (5), (2,1,1,1) and (1,1,1,1,1) each,
2/7	no of summands=2 and no of summands=3 each,
15/20	a summand to be an odd number.

K_{max} is that number $1..n$ which generates the most numerous set of partitions of n into k summands.

The set shows in its most probable state K_{max} distinct summands with respect to 1 describing dimension. A deeper discussion of the terms "distinct summands" and "distinct symbols" is outside the scope of this article.

3. Multidimensional descriptions of sets, coding and decoding of messages

3.1. Structures of levels 1 and 2

Up to now we have attached just 1 symbol on every element of the set. But among all states of the set which are identical with respect to symbol S1 further distinctions can be achieved.

Let us assume Sender has sent to Receiver a pouch containing 5 billiard balls, of which 2 carry a sign called "a". (Sign "a" can be a black dot or a red circle, its actual appearance is irrelevant, as long as it allows recognition of its existence and distinctness to the unmarked carriers.) Then Sender can employ a second symbol - called "b" - in the following fashions:

Object Nr.	1	2	3	4	5
Symbol present: "X"					
Message 1.3	X	X			
Symbol attached: "W"					
Message 2.1	X	XW			
Message 2.2	X	X	W		
Message 2.3	X	XW	W		
Message 2.4	XW	XW			
Message 2.5	X	X	W	W	

The following explanations apply:

3.1.1. First symbols

The numbering of objects is arbitrary, as Receiver shall receive the collection in one bunch, concurrently, nonsequentially, all 5 objects at the same time. The enumeration 1..5 of the objects by Receiver is arbitrary. The original message (before the applications of symbol "W") was enumerated in the table of messages as message Nr. 1.3, which means "1 symbol is present on 2 objects". Which 2 objects were pointed out by Sender by application of Symbol Nr. 1 (in the example "X") is nondecidable by Receiver.

Similarly, Receiver will not be able to decide, which of the symbols "X" or "W" is the "first" symbol.

3.1.2. Second symbol

The process of applying a second symbol of the set of symbols uses the assumptions:

nonredundant symbols are distinguishable amongst each other;

an object which carries 1 symbol can be distinguished against objects which carry 0, 2 or more symbols.

Thus, message 2.1 can be distinguished against message 1.3. The distinction happens by means of the observation: "On 5 objects, 3 carry 0 symbols, 1 carries 1 symbol - called S1 - and 1 object exists which carries 1 symbol S1 and one additional symbol, called S2." One can also formulate the observation thus: "On a collection of 5, a group of 2 exists. Within the group of 2, 2 subgroups of 1 exist".

Message 2.2 can be distinguished against both messages 1.3 and 2.1. The distinction happens thus: "On a collection of 5, exists a group of 2 and a disjunct group of 1" ("... and a group of 2 with no signs at all".) There can be 3 sender-distinct, receiver-identical messages 2.2, as Sender may distinguish which object he applies S2 to, but Receiver cannot distinguish objects 3, 4, 5 amongst each other, as he receives the carriers - as mentioned - in bulk.

Messages 2.3 and 2.5 can both as well be distinguished against the other messages in the example.

Messages 1.3, 2.2 and 2.5 show a structure of level 1, the others show a structure of level 2.

3.1.3. Redundant symbols

Message 2.4 cannot be distinguished (by Receiver) from message 1.3, as Receiver does not know whether what he sees (a conglomeration of dots, which to us appear as distinct letters X and W) is 1 symbol or an indeterminate number of individual pixels which come together.

Generally speaking: a symbol is redundant exactly in the cases:

- if the symbol is present on all objects;
- if symbol A is exactly then present if a symbol B is present, one of the symbols is receiver-indistinguishable.

3.1.4. Negated symbols

We can see in the example above that objects 3, 4 and 5 carry a - nonvisible - symbol ".not. X".

Futhermore, in message 2.1. the objects can be seen as carrying the following - nonvisible - symbols:

object 1:	".not. W",
objects 3, 4 and 5:	".not. X .and. .not. W"

In message 2.2. above, the objects carry the following - visible and nonvisible - symbols:

objects 1 and 2:	"X .and. .not. W";
object 3:	"W .and. .not. X";
objects 4 and 5:	".not. X .and. .not. W".

In message 2.4. above, the objects carry following symbols:

objects 1 and 2:	"symbol <XW>"
objects 3, 4, 5:	".not. symbol <XW>".

3.2. Identifying distinct messages

In the following two examples and the discussion relating to their distinguishability, a few new terms shall be introduced by the deictic method.

We return now to sending messages by means of media coming to the receiver in bulk. The receiver does not know, which of the distinct symbols he recongises on objects is a "first", "second" etc. symbol as he has not taken part in the coding of the message and as the symbols in themselves have no meaning.

Example A of a message identification:

On receiving the following collection:

ab, ab, ac, bc, abc

which he may as well perceive in the following form:

xy, xy, xu, yu, xyu

or in the form

\$&, \$&, \$#, &#, \$&#

or in any other forms where the group relations on 5 objects are as shown above, he may reason as follows:

- (2, 1, 1, 1) "two appear identical, 3 distinct";
- (1,1,1) "there are 3 kinds of symbols (assuming that each of them is a symbol)";
- (4, 1) "1 symbol is on 4, on 1 it isn't (assuming that {a | x | \$} is a symbol)";
- (4, 1) "1 symbol is on 4, on 1 it isn't (assuming that {b | y | &} is a symbol)";
- (3, 2) "1 symbol is on 3, on 2 it isn't (assuming that {c | u | #} is a symbol)";
- (4,1) "1 object has 3 symbols on it, 4 have 2 each".

He will furthermore observe:

- the structure depth is 3 (one element exists which is contained in 3 groups);
- the structure shallowness is 2 (every element is at least in 2 groups);
- the cardinality overstatement ratio is 11/5 (if each of the groups were exclusive, we had a set of 11 objects. We see that we have 5 objects.)

Example B of message identification:

The receiver shall of course easily distinguish from the message discussed above the following message which he receives:

S, S, S%, %, %

which he may of course perceive as follows:

q, q, qw, w, w

or in the form:

(:, (:, (:-), -, -)

or in any other forms where the group relations on 5 objects are as shown above.

His reasoning will go along following lines:

- (2, 2, 1) "twice two appear identical, 1 distinct";
- (1,1) "there are 2 kinds of symbols (assuming that each of them is a symbol)";
- (3, 2) "1 symbol is on 3, on 2 it isn't (assuming that {S | q | (:} is a symbol)";
- (3, 2) "1 symbol is on 3, on 2 it isn't (assuming that {% | w | -} is a symbol)";
- (4,1) "1 object has 2 symbols on it, 4 have 1 each".

He will furthermore observe:

- the structure depth is 2 (one element exists which is contained in 2 groups);
- the structure shallowness is 1 (every element is at least in 1 group);
- the cardinality overstatement ratio is 6/5 (if each of the groups were exclusive, we had a set of 6 objects. We see that we have 5 objects.)

3.3. Ease and efficiency of distinguishing distinct messages

The comparison of the two messages of examples A and B in the course of the decoding of the message is done by generating partitional sentences which describe the state of the set. In the examples above only one describing dimension has shown the two messages to be nondistinguishable ("(3, 2), symbol is on 3, on 2 it isn't"). Each of the other observations has immediately shown the two messages

to be distinct. The ease (efficiency) of distinguishing two distinct messages is a measure of their respective similarity.

3.4. Practical procedure of coding and decoding

In practical message transmission, Receiver will look up the collection of observed properties of the message in a table, where properties of states of sets are catalogised. This table does not necessarily physically exist as between Sender and Receiver the implicit agreement is in force that they both use as a common background the partitional properties of natural numbers, understood as descriptive logical sentences relating to structures on sets. The so-called reference table is thus an algorithm, which generates multidimensional partitions of natural numbers.

Sender relies on Receiver using the most common-sense interpretation, and Receiver relies on a common-sense (enlightened) approach towards messages transmission on the part of Sender, too.

Both Sender and Receiver rely on using the same set N and on having correctly arrived at identical results while counting distinct partitions of natural numbers. In fact, they both assume the other to have arrived at the same result relating to the most probable distribution of symbols on sets, which means also that they concurrently and congruently assume identical default parameters for following system startup values:

- length of most probable (number of objects of most efficient) message,
- most probable number of distinct symbols in most probable message,
- most probable structure depth,
- most probable structure shallowness,
- most probable number of objects carrying most, 2nd-most, 3rd-most, etc. frequently appearing symbol,
- and further probabilistic measures.

We treat the "state change" in biologic reality of a cell (which is an abstract assembly in the concept presented here) as a chemical-physiological process, which can be represented in mathematics as a change in the composition of symbols in logical sentences describing changes in states of sets. Our concept of a "most probable state" is a map of a "physiological state".

4. Relevance of the model to biologic information processing

4.1. Transmission efficiency of pointing out improbable properties

In our model, we have restricted ourselves to investigating message transmission characteristics of media coming always 3 at a time with some tricks as to their respective position {left, middle, right} which carry each any of 4 distinguishing symbols {U, C, A, G} or {A, B, C, D}.

We have tried understanding and simulating the method Nature uses and venture following proposition:

A statement, using one of 4 possibilities, which relates to a message object by means of pointing out its most improbable property (and thus implicating that the message object is with respect to the other 3 describing dimensions less improbable) imposes a restriction on N in such a fashion, that the number of message objects pointed out by this technique is less than $1/4$ of all message objects.

The truth of the above proposition can be established by counting.

One generates all possible nonredundant groups on n objects and tabulates these. One generates indexes on formal properties of the data set and transmits the names of those three describing dimensions, which in the intersection of their assertion and the negation of the union of the nontransmitted names result in the smallest set of message objects. The formal properties refer of course to the extent of deviation to the most probable distribution of objects with respect to the property.

4.2. Differentiation between "message objects" and "messages"

In the context of this article, a "message object" is represented by an element of N , a distinct case of the world to be communicated (e.g. {"the state of the ward is: n_1 patients are critical, n_2 febril,..."}, etc.). A "message" is differentiated here to a message object. Although in the von Neumann theory each message is or can be bijectively mapped to a message object, we find it convenient to name the collection of actual cases of the world the collection of message objects and distinguish this to the collection of messages. A message is a description of the state of the world and refers to 0, 1 or more message objects. A message is a distinct element of the set of possible enumerations of states of the world. In a non-encrypting practice, though, this distinction does not appear to be too relevant, as the messages serve to uniquely identify message objects. A message is here understood to mean a receiver-distinct collection of symbols on elements of a finite set. Which message object this message points to is the core question in encryption and decryption tasks.

4.3. Existence of upper limit for structures on sets

Following proposition applies:

A finite set of distinct, delineated objects can be in a finite number of distinct states.

Discussion of the proposition:

Without going through a lengthy and cumbersome proof, we offer the following plausibility considerations:

- on a finite set with $\text{card}(\text{set}) > 1$, at least 2 distinct groups can be created by application of symbols;
- the number of nonredundant groups is > 1 ;
- the number of nonredundant groups with $\text{card}(\text{set_of_objects}) \ll \text{inf}$ is also $\ll \text{inf}$;
- there exists a function $f(\text{card}(\text{set_of_objects})) \cdot \text{card}(\text{set_of_groups})$.

We state that the function referred to above has the form:

$$\text{Upper limit of distinct states of a set of } n \text{ objects} = E(n)^{\ln(E(n))}.$$

In spoken language:

A set of n objects can be in no more distinct states than the number of partitions of the cardinality of the objects raised to the power of the logarithm of the number of partitions of the cardinality of the objects.

5. Disadvantages and advantages of transmitting messages by using group structures on sets

5.1. Disadvantages

- a) both sender and receiver must operate on identical assumptions relating to fragmentational states of sets. In plain language: on both ends of the communication channel, rather powerful processors must be present;
- b) objects come in similarity classes and not every object is differentiated at every moment in the same extent. In plain language: the system focuses on specific kinds of messages and there seems to be an inertia while moving the attention to a message object with wildly differing qualities to the stream of message objects deciphered so far;
- c) theoretically, message objects can be mixed up. By taking into account only formal properties of the individual message objects, several of these can have identical addresses (be referred to by identical messages). There is a context dependency which provides unique identification. In plain language: homonyms exist (message "Mars" may refer to message objects: god, planet, sweets);
- d) the system works only with data sets of a minimal size, which are preferably prestructured and come in a quasicontinuous stream. In plain language: the number of possible symbols must be always rather finite (although they can come from an infinite multitude, at one communicational session there should be a relatively small collection of distinguishing symbols employed) and their group relations should not generate a cardinality overstatement above a limit.

5.2. Advantages

- a) any symbols can be used to generate inclusion/exclusion relations among elements of the set. In plain language: voltage or resistance changes, temporal distribution of distinct bursts, color of pixels, number of vowels and consonants, etc. can be used to the same advantage;
- b) texts in natural languages can be specifically well (efficiently) transmitted. By making use of formal properties of the text - "understanding it" - one can extremely efficiently transmit not the text but its descriptions, of which the receiver will assemble only those texts which are formally possible and very easily weed out those, which are obviously impossible (and ask back for those words which remain unintelligible);
- c) optical patterns (pictures) with their massive redundancy need not be transmitted pixel by pixel. By transmitting several descriptions of the collection, the receiver can assemble pictures easily by combinatorial means;
- d) state supervision can be achieved by minimal efforts, as the sender and receiver cooperate in shifting the respective coordinates of the most redundant message objects (which are at power-up time or at the start of communication the properties of the set N);

- e) biologic processes can be simulated by discrete automata. In plain language: there is no need to build pressurised containers with enzymes in them to use the "computational capabilities" of quasistable assemblies. It is sufficient to establish the well-defined properties of most probable states of sets and to let these oscillate around the collection of most probable values (the "physiological state").

6. Summary, outlook

We assemble the same inner picture of a car, whether we see it or whether we touch it with closed eyes or whether we hear it running. The identifying description makes use of sensation differences occurring on distinct collections of carrier media.

Coincidences of group constituting distinctions on multitudes can serve as message transmitters. This is what we said in the present article.

We have but slightly touched on following aspects of the idea:

- logical and taxonomic groundwork relating to enumerations of sets,
- standardising and codifications of implicit agreements between sender and receiver,
- bijectivity between set of logical operations and cardinality of set,
- size optimality questions,
- proposals relating to the architecture of actual processors (chips),
- probabilistic expectations of distributions of densities, of symbols and of objects.

We believe that the tool proposed and demonstrated by abstract and semantic examples can serve both for practical and for theoretical uses. The scope of the task invites further research.